Prediction of Geometry of Sonic Boom Waves Incident on Arbitrarily Oriented Plane Walls

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Two analytical methods were developed for computing the arrival time of incident and ground-reflected waves of a sonic boom acting on any given exterior wall which is exposed to the unobstructed shock wave. Method I assumed a linearly-varying speed of sound up to the tropopause and constant speed of sound above the tropopause and was considered to be the more exact method; however, it was complex and required an iterative solution for the flight altitudes above the tropopause. Method II modified method I with a simplified assumption that the ray angles in the lateral direction from the flight path were constant; the results obtained by this method compared well with the results of method I. The validity of both these methods was checked with the flight test data from the sonic boom tests conducted at Edwards Air Force Base.

Nomenclature

c = speed of sound
D_{gv} = horizontal distance between the vertex of the sonic boom and its ground intersection point g
D_{pv} = horizontal distance between a wall point p and the vertex of the sonic boom wave, at the instant p is intersected by the incident wave
M = Mach number
m = coefficient of acoustic velocity variation with altitude
S = projected distance of the wave from the flight path in the YZ plane (see Fig. 2)
T_0 = time for the wave to pass the coordinate origin after the aircraft passes over the origin
T_p = time for a wave to pass point p after the aircraft passes the coordinate origin
T_g = time for a wave to pass point g (the ground location directly under p) after the aircraft passes the coordinate origin
ΔT_p = time interval between the arrival of the incident and reflected waves at a point p on a wall
X = coordinate axis, horizontal and along the flight track on the ground
Y = coordinate axis, horizontal and perpendicular to the flight track on the ground
Z = coordinate axis, vertical
θ = angle between the wall and the Y axis, measured clockwise from Y axis in the horizontal plane
τ = time at which a pressure disturbance was emitted
ϕ = angle between the wall and the horizontal (XY) plane
β = Mach angle measured from the horizontal

Introduction

RECENT tests have indicated that sonic boom waves can cause considerable structural damage to structures. To predict the structural response due to the passage of a sonic boom wave, it is necessary to know the wave/structure geometry. In this paper, two methods were developed for calculating the time of arrival of an incident sonic boom wave and its ground-reflected wave on a wall, as functions of the aircraft speed and altitude, the wall's slope and its distance and the angle from the flight track. Once the times-of-arrival of the wave are known at several points of the wall, the geometric relationship between the wave and the wall can be found. Then the structural response can be predicted, if the strength of the sonic boom wave is known. In this paper, no attempt is made to predict the strength of the sonic boom wave. This problem has already been extensively discussed in the literature. In a recent report,[1] an analysis along with a computer program to predict the sonic boom pressure signatures as functions of weather information, altitude, information on the flight path, and aircraft geometry was developed.

In method I, a ballistic wave approach[2,3] was used, with a realistic atmospheric temperature-altitude relation. In the absence of relevant weather information this method can be considered as an exact method. However, this method was very complex and required an iterative solution for flight altitudes above the tropopause.

In method II, a simplified assumption of treating the ray angles as constant in the lateral direction from the flight track was adopted and the solution was a closed-form analytical expression. The results obtained by both these methods compare very well.

Geometry of a Sonic Boom Wave Incident on a Plane Wall

The times of arrival of incident and reflected sonic boom waves at any point on a wall exposed to the unobstructed shock wave depend on the geometric relations between the wave and wall, for a given flight altitude, direction, and speed of an aircraft. The coordinate axis system for an arbitrarily oriented plane wall is shown in Fig. 1 and the model of a ballistic wave intersecting the wall is shown in Fig. 2.

If the time of arrival of a plane weak wave is known at three noncolinear points on a wall, the angular relationship between the wall and wave can be calculated. The bow and tail waves were assumed to have been produced by an aircraft in steady, level flight at altitude Z_v, flying with a velocity

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parallel to the X axis. In this analysis, only bow waves and their ground-reflected waves were considered. For a known geometry and forward speed of the aircraft, the arrival time of an incident wave and the time interval between the arrival of an incident wave and its ground-reflected wave at a given point p on the wall can be expressed as follows:

$$T_p = \frac{X_v}{V} = \frac{1}{V}(X_p + D_{pv}) = \frac{1}{V}\left[(Y_p - Y_c)\tan\theta + Z_p\left(\frac{\cot\phi}{\cos\theta}\right) + D_{pv}\right] \quad (1)$$

$$\Delta T_p = (2T_g - T_p) = (2/V)(D_{gv} - D_{pv}) \quad (2)$$

The only unknowns in Eqs. (1) and (2) are the distances D_{gv} and D_{pv}. In the following sections, two methods are described to predict these values.

Method I

This method was based on the ballistic wave analysis[1,2] with the assumption that the speed of sound decreases linearly up to the tropopause (assumed as 36,000 ft), and is constant above the tropopause. The equation of the speed of sound up to the tropopause can be expressed as

$$c = c_g - mZ \quad (3)$$

For the values of $c_g = 1116$ fps and $m = 0.004$ fps-ft, the speed of sound variation with altitude was found to be very near that of the standard atmosphere.

Flight Altitudes below the Tropopause

The shape of the wave fronts produced by a point disturbance was obtained from the system of surfaces which are orthogonal to rays from that point. By following that disturbance as it propagates from a point along a given ray, it is possible to relate the shape of the wave fronts to the growth of these fronts. In an atmosphere in which the sonic speed decreases linearly with altitude, the disturbance front at any time t, emitted at a disturbance origin d, at an earlier time τ, can be expressed as

$$(X - X_d)^2 + (Y - Y_d)^2 + \{Z - Z_d + (c_d/m)[\cosh m(t-\tau) - 1]\}^2 = (c_d/m)^2 \sinh^2 m(t-\tau) \quad (4)$$

where $c_d = c_g - mZ_d$, and X_d, Y_d, and Z_d are functions of τ.

The envelope of this system of wave fronts is obtained by eliminating τ between Eq. (4) and its partial derivative with respect to τ. The equation obtained by this differentiation, when combined with the steady and level flight conditions for the chosen coordinate system (see Fig. 2) yields

$$(X - X_d) = -(1/M)(Z - Z_v - c_v/m)\sinh m(t-\tau) \quad (5)$$

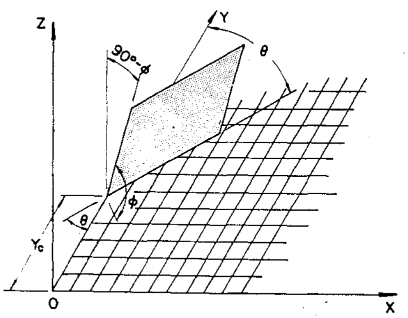

Fig. 1 Coordinate axis system for an arbitrarily oriented plane wall.

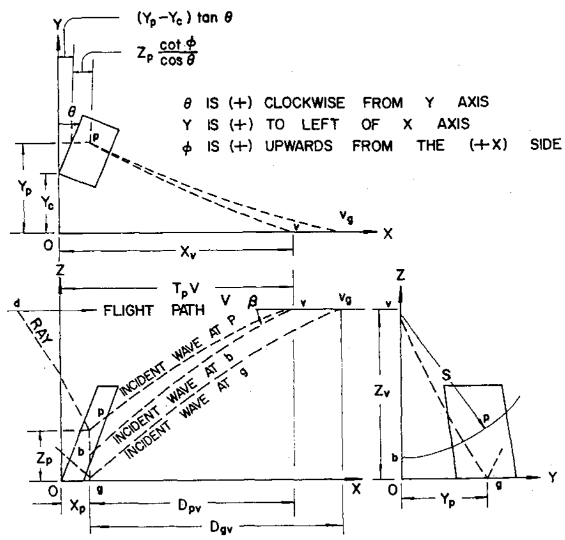

Fig. 2 Model for the analysis of a ballistic wave intersecting a plane, rectangular, sloping wall.

Equations (4) and (5) are combined and simplified to yield the relation

$$(1/M^2)(Z - Z_v - c_v/m)^2 \cosh^2 m(t-\tau) + (2c_v/m)(Z - Z_v - (c_v/m))\cosh m(t-\tau) + Y^2 + (Z - Z_v - c_v/m)^2(1 - 1/M^2) + (c_v/m)^2 = 0 \quad (6)$$

The roots of the equation are contained in

$$\cosh m(t-\tau) = M/(Z - Z_v - c_v/m) - Mc_v/m \pm \{(1 - 1/M^2)[(Mc_v/m)^2 - (Z - Z_v - c_v/m)^2] - Y^2\}^{1/2} \quad (7)$$

For positive values of Z and Z_d, the (+) sign applies before the radical in Eq. (7).

For given values of $Y = Y_p$, $Z = Z_p$, Z_v, and M $(t-\tau)_p$ can be evaluated by Eq. (7). The time elapsed between the passing of the aircraft over the origin 0 and incident wave arrival at point p can be expressed as (see Fig. 2)

$$T_p = (1/V)(X_p + D_{pv}) = (1/V)[(Y_p - Y_c)\tan\theta + Z_p(\cot\phi/\cos\theta)] + (t-\tau)_p - (X_p - X_d)/V \quad (8)$$

The time interval between incident and reflected waves can be expressed as

$$\Delta T_p = (2/V)(D_{gv} - D_{pv}) = (2/V)\{[(t-\tau)_p V - (X_p - X_d)] - [(t-\tau)_g V - (X_g - X_{dg})]\} \quad (9)$$

The $(X_g - X_d)$ and $(t-\tau)_g$ terms can be computed using Eqs. (5) and (7), respectively, for the values of $Y = Y_g$ and $Z = Z_g$.

Flight Altitudes above the Tropopause

For flight altitudes above the tropopause, an explicit solution is not possible for a given offset distance from the flight track. An iteration method was developed for computing the local arrival times of incident and reflected waves of a sonic boom wave.

For $Z_v \geq Z \geq Z_t$, where Z_t is the altitude of the tropopause, the speed of sound is assumed to be constant. The disturbance envelope at time t for a point disturbance emitted by the aircraft at τ is a cone,

$$(X - X_d)^2 + (Y - Y_d)^2 + (Z - Z_d)^2 = c_d^2(t-\tau)^2 \quad (10)$$

Differentiation of Eq. (10) with respect to τ and the application of steady and level flight conditions, gives

$$(X - X_d) = c_v(t-\tau)/M \quad (11)$$

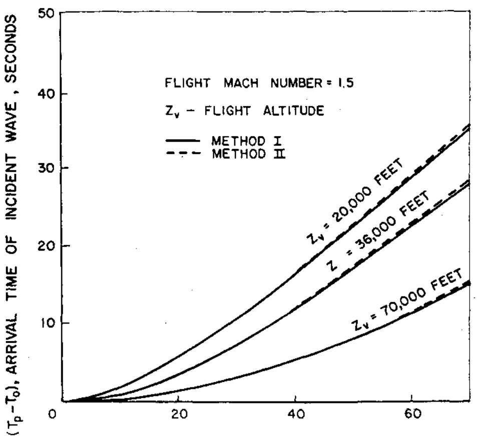

Fig. 3 Comparison of incident wave arrival times by method I and method II.

Equations (10) and (11) are combined and the result is

$$Y^2 + (Z - Z_v)^2 = c_v^2(t - \tau)^2(1 - 1/M^2) \quad (12)$$

Differentiation of Eq. (12) with respect to τ and recognition of the orthogonal property between any ray and the disturbance envelope gives

$$(\partial Y/\partial Z)|_{\text{ray}} = \text{const} = Y/(Z - Z_v) \quad (13)$$

When the tropopause conditions $Y = Y_t$ and $Z = Z_t$ are substituted into Eqs. (12) and (13), the resulting expressions are

$$Y_t^2 + (Z_t - Z_v)^2 = c_v^2(t - \tau)^2(1 - 1/M^2) \quad (14)$$

and

$$(\partial Y/\partial Z)|_{\text{ray}} = Y_t/(Z_t - Z_v) \quad (15)$$

For $Z_t \geq Z \geq 0$, the cone is warped by the temperature gradients and no exact analysis is available. For local wave positions and arrival times, however, it is sufficient to consider only the ray which reaches the given point. If the ray at the tropopause plane is considered to be a point disturbance, the previously developed equations will describe its path. Then the location of the wave can be found as a function of time, working point by point with a ray tracing procedure.

The equation for the disturbance envelope at time t of a point disturbance emitted at time t_t at the tropopause plane is

$$\cosh m(t - t_t) = [M/(Z - Z_t - c_t/m)]\{(-Mc_t/m) + [(1 - 1/M^2)(Mc_t/m)^2 - (Z - Z_t - c_t/m)^2 - (Y - Y_t)^2]^{1/2}\} \quad (16)$$

Equation (16) can be solved as before, except that now the value of Y_t is not known. An iterative method must be used. 1) A value is assumed for Y_t. 2) t_t and $\partial Y/\partial Z|_{\text{ray}}$ at the tropopopause are calculated by Eqs. (14) and (15). 3) The slope is assumed to be constant for a selected Z interval,

$$Y - Y_t = (\partial Y/\partial Z)\Delta Z, Z = Z_t - \Delta Z \quad (17)$$

4) $(t - t_t)$ is calculated from Eq. (16) by the techniques explained earlier. The new wave slopes are calculated as follows. $\partial Y/\partial Z$ can be evaluated from Eq. (6) for the disturbance envelope which is everywhere perpendicular to the rays. For a point disturbance at the point (X_t, Y_t, Z_t), Eq. (6) is differentiated with respect to Z, after replacing Y with $Y - Y_t$, and the resulting expression is

$$2/M^2 (Z - Z_t - c_t/m) \sinh^2 m(t - t_t) + 2(Y - Y_t)(\partial Y/\partial Z)|_{\text{dist. envel.}} + 2(Z - Z_t - c_t/m) + 2(c_t/m) \cosh m(t - t_t) = 0 \quad (18)$$

Since the disturbance and the ray are orthogonal,

$$(\partial Y/\partial Z)|_{\text{dist. envel.}} (\partial Y/\partial Z)|_{\text{ray}} = -1 \quad (19)$$

Equations (18) and (19) are simplified and the result is

$$\left.\frac{\partial Y}{\partial Z}\right|_{\text{ray}} = (Y - Y_t) \Bigg/ \left[\left(Z - Z_t - \frac{c_t}{m}\right) \times \left(1 + \frac{\sinh^2 m(t - t_t)}{M^2}\right) + \frac{c_t}{m} \cosh m(t - t_t)\right] \quad (20)$$

5) Steps 3 and 4 are repeated until the desired Z value is obtained. If the computed Y does not give the desired location Y_p, a new Y_t is selected and the procedure is repeated till the desired value is obtained. 6) When the desired Y location is obtained to sufficient accuracy, the T_p value is computed by Eq. (8) with the new intervals,

$$(t - \tau)_p = (t_t - \tau)_p + (t - t_t)_p \quad (21)$$

and

$$(X_p - X_d) = (X_p - X_t) + (X_t - X_d) \quad (22)$$

For the proper Y_t value, Eqs. (21) and (22) can be solved from Eqs. (14, 16, and 5) and the relation,

$$(X_t - X_d) = (c_t/m)(t_t - \tau)_p \quad (23)$$

7) For the g location, steps 1–5 must be performed again, with all p subscripts replaced by g. D_{gv} can now be found with $(t - \tau)_g$ and $(X_g - X_{dg})$ as was done for p in step 6. 8) ΔT_p is computed with Eq. (9).

This method is considered to be the most accurate prediction method available without specific meteorological information along the wave path for each flight.

Method II

A second method was developed to find the arrival times by an explicit relation even for flight altitudes above the tropopause. The simplifying assumption added was that dY/dZ of the ray was constant at the value of the flight altitude, while the linear variation of speed of sound with altitude below the tropopause was retained. The time of arrival of an incident wave is then given by

$$T_p = \frac{1}{V}(X_p + D_{pv}) = \frac{1}{V}\left[X_p + \int_0^{S_p} \frac{dS}{\tan\beta}\right] \quad (24)$$

Table 1 Comparison of the theoretical arrival times of sonic boom waves with the flight test data

Flight Mach number	Flight altitude, ft	Lateral distance, ft	T_{31}, sec Method II	T_{31}, sec Flight testing	T_{56}, sec Method II	T_{56}, sec Flight testing
2.50	59,400	0	0.0840	0.0840	0.0159	0.0155
2.50	60,300	182	0.0820	0.0820	0.0161	0.0155
2.46	59,700	13,376	0.0818	0.0770	0.0157	0.0150
2.51	59,400	31,616	0.0851	0.0810	0.0135	0.0135
2.49	59,100	68,096	0.0871	0.0895	0.0085	0.0065
2.50	60,000	68,096	0.0868	0.0940	0.0083	0.0070
2.50	60,000	68,096	0.0808	0.0830	0.0089	0.0080
2.50	60,300	71,136	0.0781	0.0800	0.0081	0.0100
1.80	60,300	61	0.1164	0.1155	0.0139	0.0125
1.80	59,700	182	0.1194	0.1195	0.0138	0.0125
1.80	59,700	730	0.1150	0.1145	0.0141	0.0120
1.80	60,000	6,384	0.1133	0.1140	0.0136	0.0120
1.80	60,600	9,728	0.1133	0.1150	0.0138	0.0130
1.50	38,600	0	0.1355	0.1440	0.0122	...
1.50	37,600	912	0.1446	0.1455	0.0110	0.0100
1.46	37,200	10,336	0.1475	0.1480	0.0098	0.0090
1.48	37,300	37,696	0.1453	0.1600	0.0041	...

S is the projected distance of the wave from the flight path in the YZ plane (see Fig. 2), and

$$dS = [(dY^2 + dZ^2)]^{1/2} = -dZ[1 + (dY/dZ)^2]^{1/2} \quad (25)$$

Flight Altitudes below the Tropopause

The assumptions and geometry give the following:

$$(dY/dZ)|_{\text{ray}} = \text{const} = (Y - Y_v)/(Z - Z_v),\ Y_v = 0$$
$$dS = -dZ\{1 + [Y/(Z - Z_v)]^2\}^{1/2}$$
$$c = c_g - mZ,\ dZ = -dc/m$$

and $\quad (26)$

$$\tan\beta = \frac{1}{(M^2 - 1)^{1/2}} = \frac{c}{(V^2 - c^2)^{1/2}}$$

Equations (1, 24, 25, and 26) are combined and the resulting expressions for T_p and T_g are

$$T_p = \frac{1}{V}\left[(Y_p - Y_c)\tan\theta + Z_p\left(\frac{\cot\phi}{\cos\theta}\right)\right] + \frac{1}{mV}\left[1 + \left(\frac{Y_p}{Z_p - Z_v}\right)^2\right]^{1/2}\left[(V^2 - c_p^2)^{1/2} - (V^2 - c_v^2)^{1/2} + V\ln\left(\frac{V + (V^2 - c_v^2)^{1/2}c_p}{V + (V^2 - c_p^2)^{1/2}c_v}\right)\right] \quad (27)$$

and

$$T_g = \frac{1}{V}\left[(Y_p - Y_c)\tan\theta + Z_p\left(\frac{\cot\phi}{\cos\theta}\right)\right] + \frac{1}{mV}\left[1 + \left(\frac{Y_p}{Z_p}\right)^2\right]^{1/2}\left[(V^2 - c_g^2)^{1/2} - (V^2 - c_v^2)^{1/2} + V\ln\left(\frac{V + (V^2 - c_v^2)^{1/2}c_g}{V + (V^2 - c_v^2)^{1/2}c_g}\right)\right] \quad (28)$$

From Eqs. (2, 27, and 28), ΔT_p can be computed.

Flight Altitudes above the Tropopause

The assumption of constant dY/dZ eliminates the iterative solution which was required in method I. The time of arrival of an incident wave can be expressed as

$$T_p = \frac{1}{V}\left[X_p + \int_0^{S_p}\frac{dS}{\tan\beta}\right] = \frac{1}{V}\left[X_p + \int_0^{S_t}\frac{dS}{\tan\beta} + \int_{S_t}^{S_p}\frac{dS}{\tan\beta}\right] \quad (29)$$

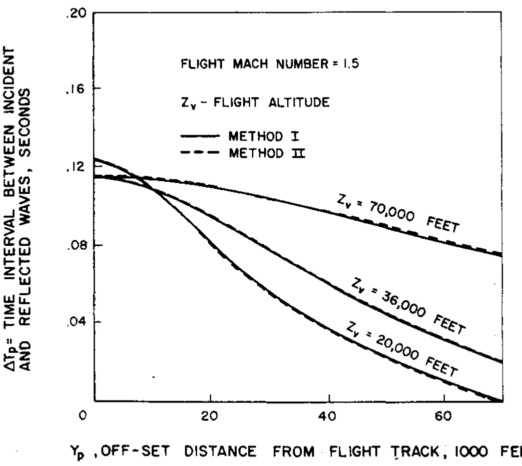

Fig. 4 Comparison of the time intervals between incident and reflected waves by method I and method II.

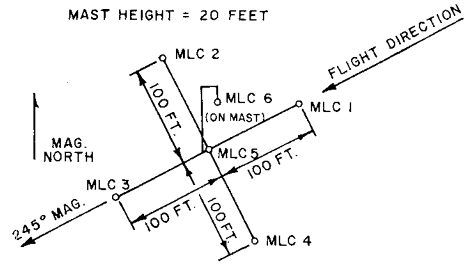

Fig. 5 Free-field microphone cruciform array.

In view of the assumptions, c = const in the region $Z_v \geq Z \geq Z_t$,

$$Y_t/(Z_t - Z_v) = \text{const} = Y_p/(Z_p - Z_v) \quad (30)$$

the integral terms in Eq. (29) become

$$\int_0^{S_t}\frac{dS}{\tan\beta} = \left[\frac{Z_v - Z_t}{c_v}\right](V^2 - c_v^2)^{1/2}\left[1 + \left(\frac{Y_t}{Z_t - Z_v}\right)^2\right]^{1/2}$$

and

$$\int_{S_t}^{S_p}\frac{dS}{\tan\beta} = \frac{1}{m}\left[1 + \left(\frac{Y_p}{Z_p - Z_v}\right)\right]^{1/2} \times \left\{(V^2 - c_p^2)^{1/2} - (V^2 - c_v^2)^{1/2} + V\ln\left(\frac{V + (V^2 - c_v^2)^{1/2}c_p}{V + (V^2 - c_p^2)^{1/2}c_v}\right)\right\} \quad (31)$$

Equations (1, 29, 30, and 31) are combined and the resulting expressions for T_p and T_g are

$$T_p = \frac{1}{V}\left[(Y_p - Y_c)\tan\theta + Z_p\left(\frac{\cot\phi}{\cos\theta}\right)\right] + \frac{1}{V}\left[1 + \left(\frac{Y_p}{Z_p - Z_v}\right)^2\right]^{1/2}\left\{\left(\frac{Z_v - Z_t}{c_v}\right)(V^2 - c_v^2)^{1/2} + \frac{1}{m}(V^2 - c_p^2)^{1/2} - \frac{1}{m}(V^2 - c_v^2)^{1/2} + \frac{V}{m}\ln\left(\frac{V + (V^2 - c_v^2)^{1/2}c_p}{V + (V^2 - c_p^2)^{1/2}c_v}\right)\right\} \quad (32)$$

and

$$T_g = \frac{1}{V}\left[(Y_p - Y_c)\tan\theta + Z_p\left(\frac{\cot\phi}{\cos\theta}\right)\right] + \frac{1}{V}\left[1 + \left(\frac{Y_p}{Z_p}\right)^2\right]^{1/2}\left\{\left(\frac{Z_v - Z_t}{c_v}\right)(V^2 - c_v^2)^{1/2} + \frac{1}{m}(V^2 - c_g^2)^{1/2} - \frac{1}{m}(V^2 - c_v^2)^{1/2} + \frac{V}{m}\ln\left(\frac{V + (V^2 - c_v^2)^{1/2}c_g}{V + (V^2 - c_g^2)^{1/2}c_v}\right)\right\} \quad (33)$$

From Eqs. (2, 32, and 33), ΔT_p can be computed.

Method II is much more straightforward than method I and does not require any iterative solution. However, the assumption of constant ray angles in lateral direction, places its accuracy in doubt. Therefore, a number of typical cases were computed by both the methods and these methods were successfully applied to practical problems.[4,5]

Results

In order to compare the two methods, computations were performed for a vertical wall location of 100 ft above ground level, which was assumed to be at sea level. Values of $(T_p - T_0)$ and ΔT_p were computed for several flight altitudes and Mach number range and offset distances. The

T_p and ΔT_p results of both the methods for flight Mach number 1.5 and flight altitude of 70,000, 36,000, and 20,000 ft were compared (see Figs. 3 and 4). Method II, in spite of its simplified assumption of constant ray angles, gave results which are in very good agreement with the results of method I. Results for flight Mach numbers 2.0 and 3.0 by the two methods were in even closer agreement. Also, method II was tested against the flight test data of sonic boom experiments conducted at Edwards Air Force Base.[6] The microphone layout used at Edwards Air Force Base to record flight test data was shown in Fig. 5 and the comparison of the results of method II and flight tests was given in Table 1. The results show excellent agreement for the flight direction, as would be expected. For T_{56}, the interval of arrival time at two points vertically separated, the computed values were generally too large. This indicates that the wave front angle was greater than the prediction and may be due to greater atmospheric temperature gradients or to prevailing wind gradients. The average error was 10.6%.

Conclusions

In the absence of better meteorological information, either of the methods are very effective tools to predict the arrival times of incident and reflected waves on arbitrarily oriented plane walls. However, method II, with its simplified assumption of constant ray angles in lateral direction, is very simple for computation and is recommended.

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